Statistics

Lecture 8

Feb 19-8:47 AM

In-Person QZ 6

Given $\boxed{P(E) = .6}$

1) Find $P(\bar{E}) = 1 - P(E) = \boxed{.4}$ ✓

2) odds in favor of event E.

$P(E) : P(\bar{E})$ $\quad .6 : .4 \Rightarrow \boxed{3:2}$ ✓

.6 $\boxed{\div}$ .4 $\boxed{\text{MATH}}$ $\boxed{\text{1: ▸Frac}}$ $\boxed{\text{Enter}}$ $\frac{3}{2}$

3) odds against event E.

$\boxed{2:3}$ ✓

Apr 12-10:39 AM

If we have n different items, and we wish to select r of them in any order without replacement, we can do it in

$_nC_r$ number of ways.

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$

7 different items
we want to select
2 of them
No replacement,
order does not matter

$$_7C_2 = \frac{7!}{2!\cdot(7-2)!} = \frac{7!}{2!\cdot 5!} = \frac{7\cdot \cancel{6}^{3}\cdot \cancel{5!}}{\cancel{2}\cdot 1\cdot \cancel{5!}} = \boxed{21}$$

# of ways this can be done is given by $_7C_2$

Using TI

7 $\boxed{\text{MATH}}$ $\boxed{\rightarrow}$ PRB ↓ $\boxed{3{:}\,_nC_r}$ 2 $\boxed{\text{Enter}}$

$\boxed{21}$

Apr 19-8:06 AM

A basketball team has 15 players and 5 players needed to start a game.
Assume every player can play any position.
How many ways can we select 5 players to start the game?

$$_{15}C_5 = \boxed{3003}$$

15 $\boxed{\text{MATH}}$ $\boxed{\rightarrow}$ PRB $\boxed{\downarrow}$ $\boxed{3{:}\,_nC_r}$ 5 $\boxed{\text{Enter}}$

5 females and 8 males.
We need to select 1 female and 2 males.
How many ways can this be done if order does not matter and no replacement?

# of ways for females · # of ways for males

$$_5C_1 \cdot {}_8C_2 = \boxed{140}$$

What about 2 females and 1 male?

$$_5C_2 \cdot {}_8C_1 = \boxed{80}$$

Apr 19-8:12 AM

A box has 4 Red, 6 white, and 10 blue balls.
we wish to take 3 balls randomly.
order does not matter, No replacement.

1) How many ways can this be done?

$$_{20}C_3 = 1140$$

2) How many ways can we have one of each color?

$$_4C_1 \cdot {}_6C_1 \cdot {}_{10}C_1 = 240$$

3) $P(\text{Selecting one of each Color}) = \dfrac{_4C_1 \cdot {}_6C_1 \cdot {}_{10}C_1}{_{20}C_3} = \dfrac{240}{1140} = \boxed{\dfrac{4}{19}}$

odds in favor of selecting one of each Color 4 : 15

odds against 15 : 4

Apr 19-8:21 AM

4) P(at least 1 white color ball)

$$= 1 - P(\text{No white color Ball})$$

$$= 1 - \frac{_6C_0 \cdot {}_{14}C_3}{_{20}C_3} = \boxed{\frac{194}{285}}$$

5) P(at least 1 red color ball)

$$= 1 - P(\text{No red color ball})$$

$$= 1 - \frac{_4C_0 \cdot {}_{16}C_3}{_{20}C_3} = \frac{29}{57}$$

Apr 19-8:29 AM

A standard deck of playing cards has 52 Cards, 26 Red, 12 Face, and 4 Aces.

Draw 5 Cards, No replacement, order does not matter.

P(exactly 2 face and 2 Aces) $= \dfrac{{}_{12}C_2 \cdot {}_{4}C_2 \cdot {}_{36}C_1}{{}_{52}C_5}$

12 face + 4 Aces + $\boxed{\text{36 other Cards}}$

$= \dfrac{14256}{2598960} \approx .005$

even

Divide by 2

$= \dfrac{7128}{1299480} = \dfrac{3564}{649740} = \dfrac{1782}{324870} = \boxed{\dfrac{297}{54145}}$

Apr 19-8:38 AM

SG 14 & 15

Data
- 1) Numerical
  - 1) Discrete — Countable
  - 2) Continuous — Measureable
- 2) Non-Numerical

Let x be a discrete random Variable with prob. dist. of $P(x)$.

Prob. dist. is a method that gives the Prob. of all possible outcomes.

It could be
1) in the form of a chart / table.
2) in the form of a graph
3) in the form of a formula or just concept of prob.

Apr 19-9:11 AM

Some rules

1) $0 \leq P(x) \leq 1$

2) Sum of $P(x)$ is always 1.

3) $P(x) = 1 \longleftrightarrow$ Sure event

4) $P(x) = 0 \longleftrightarrow$ Impossible event

5) $0 < P(x) \leq .05 \longleftrightarrow$ rare events

Apr 19-9:16 AM

Consider the chart below for disc. random Variable x with prob. dist. $P(x)$.

x	$P(x)$
1	.2
2	.5
3	.3

1) Verify $\sum P(x) = 1$ ✓

$.2 + .5 + .3 = 1$ ✓

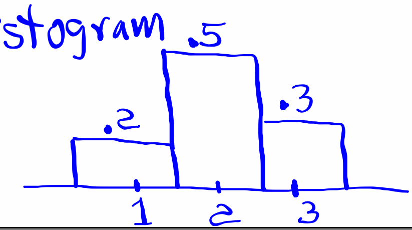
2) $P(x = 1 \text{ or } x = 3)$

$= .2 + .3 = \boxed{.5}$

3) Draw Prob. dist. histogram

$x \to$ Midpoints

$P(x) \to$ Rel. Freq.



Apr 19-9:19 AM

Consider the chart below for discrete random variable x with prob. dist. $P(x)$

x	$P(x)$
1	.2
2	.3
3	.4
4	.1

1) Find $P(x=4)$

$= 1 - [.2 + .3 + .4] = \boxed{.1}$

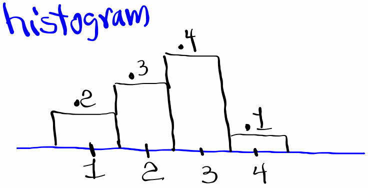
2) Find $P(x=1 \text{ or } x=4)$

$= .2 + .1 = \boxed{.3}$

3) Find $P(x \le 3) = .4 + .3 + .2 = \boxed{.9}$

4) Find $P(x \ge 2) = .3 + .4 + .1 = \boxed{.8}$

5) Draw Prob. dist. histogram



Apr 19-9:24 AM

Complete the chart below

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.3	.3	.3
2	.5	1.0	2.0
3	.2	.6	1.8

1) $\sum xp(x)$

$= \boxed{1.9}$

2) $\sum x^2 p(x)$

$= \boxed{4.4}$

3) Compute $\sum x^2 p(x) - \left(\sum xp(x)\right)^2$

$= 4.4 - 1.9^2 = \boxed{.49}$

4) Find $\sqrt{\text{Last answer}} = \sqrt{.49} = \boxed{.7}$

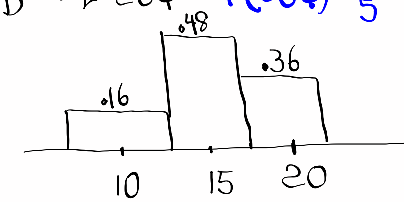
Apr 19-9:31 AM

2 Nickels, 3 Dimes, take 2 Coins with replacement

NN
ND
DN
DD
} Sample Space

	Total	
NN →	10¢	$P(10¢) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$
ND, DN →	15¢	$P(15¢) = 2\left(\frac{2}{5} \cdot \frac{3}{5}\right) = \frac{12}{25}$
DD →	20¢	$P(20¢) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$

Total	P(Total)
10	.16
15	.48
20	.36



P(Total of 10¢ or 20¢)

$= .16 + .36 = \boxed{.52}$

Apr 19-9:37 AM

Mean μ mu

Variance σ^2 Sigma²

Standard dev. σ Sigma

$$\boxed{\begin{aligned} \mu &= \sum x p(x) \\ \sigma^2 &= \sum x^2 p(x) - \mu^2 \\ \sigma &= \sqrt{\sigma^2} \end{aligned}}$$

Apr 19-9:44 AM

x (L1)	P(x) (L2)	$x\,P(x)$	$x^2 P(x)$
1	.2	.2	.2
2	.3	.6	1.2
3	.4	1.2	3.6
4	.1	.4	1.6

$\mu = \sum x\,p(x) = .2 + .6 + 1.2 + .4 = \boxed{2.4}$

$\sigma^2 = \sum x^2 p(x) - \mu^2 = .2 + 1.2 + 3.6 + 1.6 - 2.4^2 = \boxed{.84}$

$\sigma = \sqrt{\sigma^2} = \sqrt{.84} \approx .917$

Using TI

$x \to L1$, $P(x) \to L2$

STAT → CALC

1: 1-Var Stats

List: L1

Freqlist: L2

Calculate

No Menu L1, L2 enter

$\mu = \bar{x} = 2.4$

$\sigma = \sigma_x = .917$

$n = 1$ ← Total Prob.

VARS 5: Statistics

4: σ_x x^2 Enter

.84

Math 1: ▶Frac Enter

$\frac{21}{25}$

Apr 19-9:47 AM

Consider the chart below

x	P(x)
1	.05
2	.15
3	.25
4	.35
5	.20

1) $P(X=5)$

$= 1 - [.05 + .15 + .25 + .35]$

$= \boxed{.2}$

$x \to L1$, $P(x) \to L2$

Use 1-Var Stats with L1 & L2

$\mu = 3.5$

$\sigma = 1.118$

$n = 1$

VARS

5: Statistics

4: σ_x x^2

MATH 1: ▶Frac

Enter

$\frac{5}{4}$

S_x blank

Apr 19-9:58 AM

68% Range $\Rightarrow \mu \pm \sigma$

95% Range $\Rightarrow \mu \pm 2\sigma$

Usual Range

99.7% Range $\Rightarrow \mu \pm 3\sigma$

x	$P(x)$
10	.7
30	.2
50	.1

1) Verify $\sum P(x) = 1$.

$x \rightarrow L1$, $P(x) \rightarrow L2$

Use 1-Var Stats with L1 & L2

$\mu = 18$

$\sigma = 13.266$

$n = 1$

$\sigma^2 = 176$

Reduced fraction

S_x Blank

$\mu = 18$, $\sigma \approx 13$

68% Range

$\mu \pm \sigma \rightarrow 18 \pm 13 \rightarrow \boxed{5 \text{ to } 31}$

Apr 19-10:05 AM

Pay me \$10, 24 students

we have a drawing.

If your name is drawn, I give you

a TI-84 Calc.

I collect $10(24) = \$240$

Calc. is worth \$120

my Net profit \$120

\$120/24 TKts

= \$5

I make \$5

Per ticket

Expected Value

Per ticket

Net	P(Net)	
10 − 120	1/24	I lose
10 − 0	23/24	I win

Net $\rightarrow x \rightarrow$ L1

P(Net) $\rightarrow P(x) \rightarrow$ L2

Use 1-Var Stats with L1 & L2

E.V. = $\mu = \bar{x}$

\$5

Apr 19-10:24 AM

Pay me $5
Draw one card from a standard deck of playing cards

If you draw	I give you
Ace	$20
Face	$10
Any other card	$0

L1 L2

Net	P(Net)	
5 - 20	4/52	Ace
5 - 10	12/52	Face
5 - 0	36/52	any other card

Find E.V. per bet for the house

$1.15 / bet

Apr 19-10:31 AM

You buy luggage insurance for $100

Any damages to your luggage, airline pays you $1000.

P(damage) = .1%

Find E.V. for each policy sold.

Net	P(Net)	
100 - 1000	.1% = .001	Damage
100 - 0	99.9% = .999	~~Damage~~

Use $\boxed{\text{1-Var Stats}}$ with L1 & L2

$E.V. = \mu = \bar{x}$

$99

Net → L1
P(Net) → L2

Find σ^2 in reduced frac.

999

Apr 19-10:36 AM

3 Quarters

7 Dimes

Take 2 Coins

No replacement

Total	P(Total)
20	56/90
35	28/90
50	6/90

DD → 20¢ → $P(20¢) = \frac{7}{10} \cdot \frac{6}{9} = \frac{56}{90}$

DQ

QD → 35¢ → $P(35¢) = \qquad = \frac{28}{90}$

QQ → 50¢ → $P(50¢) = \frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90}$

$\mu = 26.\overline{6}$

$\sigma = 9.250$

$n = 1$

σ^2 exact

$\boxed{\frac{770}{9}}$

Apr 19-10:43 AM